J. Fluid Mech. (1996), vol. 317, pp. 407–412

REVIEWS

Turbulence: The Legacy of A. N. Kolmogorov. By U. FRISCH. Cambridge University Press, 1995. 296 pp. ISBN 0 521 45713 0. £15.95.

The legacy of Kolmogorov provides specialists with a strong incentive to write another book on turbulence, even 54 years after the publication of the celebrated K41s.

This book consists of two essentially different parts with some cross-references: the main text (chapters 1–8), and a guided tour to further reading (chapter 9) which occupies about a quarter of the book. The first part is largely based on the author's lectures in Les Houches (1981), Varenna (1983), Boulder (1987), and the paper in the Kolmogorov issue of *Proceedings of the Royal Society, London* (1991) and is mainly concerned with aspects of homogeneous and isotropic turbulence with an emphasis on scaling. The second part is a critical review of a large amount of material beyond the scope of the first *textbook* part. In spite of this, the statement in the book Prospectus that "This textbook presents a modern account of turbulence" is too broad a characterization for a textbook of 300 pages only, and may be misleading for an unwary and/or uninformed reader. The character of the book lies in the range of problems which reflect the author's interests and beliefs.

The book starts with simple examples illustrating successive breaking of different flow symmetries as the Reynolds number increases. This is followed by a kind of definition: "Turbulence at very high Reynolds numbers when all or some of the possible symmetries are restored in the statistical sense, is known as fully developed turbulence". Chapter 2 is devoted to symmetries of the Navier–Stokes equations (space and time translations, Galilean transformations, the full group of rotations, which include reflections, and scaling), conservation laws and the scale-by-scale energy budget equation.[1] In chapter 3 dynamical systems are used to justify the probabilistic description of turbulence. Kolmogorov has stated on this matter: "From the very beginning it was clear that the theory of random functions of many variables (random fields) whose development only started at that time must be the underlying mathematical technique" (*Selected Works of A. N. Kolmogorov, Vol. I*, ed. V. M. Tikhomirov, p. 487, Kluwer, 1991). Although the author makes some cautionary remarks on the use of dynamical systems this seems hardly appropriate. Methods of dynamical systems theory, after an initial period of euphoria and even claims that the problem of turbulence was solved, have proved to be ineffective/irrelevant for the theory of fully developed turbulence. Quoting G. K. Batchelor 1989: "...considerations of the properties of fully developed turbulence require rather different ideas,..." (*J. Fluid Mech.* vol. 205, 1989, p. 593).

A brief review of some probabilistic tools is given in chapter 4, and a cautious exposition of two experimental laws of fully developed turbulence in chapter 5, viz. the two-thirds law (and the corresponding five-thirds energy spectrum), and the energy dissipation law for decreasing viscosity, which "behaves in a way consistent with a finite positive limit". The presentation of the Kolmogorov 1941 theory is given in chapter 6. In the introduction to this chapter it is stated that "a modern view point with emphasis on postulated symmetries rather than on postulated universality, i.e. independence of the particular mechanism by [which – A.T.] the turbulence is generated". This is done via formulation of three hypotheses for the limit of infinite Reynolds numbers, at small scales and away from boundaries: (H1) all possible symmetries of NSE are restored in the statistical sense; (H2) there is global self-similarity, i.e. the scaling exponent is unique; (H3) the dissipation rate is finite, which fixes this exponent as 1/3. A special feature of this chapter is the careful and detailed derivation of the Kolmogorov four-fifths law and related matter. It is the first complete derivation of four-fifths law in the literature (but see V. Yakhot, 'Spectra of fluctuations of velocity, kinetic energy and the dissipation rate in strong turbulence', *Phys. Rev.* E. vol. 50, 1994, pp. R20–R23 and references therein). The chapter concludes with discussion of the lack of universality in view of the famous Landau remarks and objections.

There are also disappointing aspects of the author's presentation of the K41 theory. In the first K41 paper before putting forward his two similarity hypotheses Kolmogorov formulated the hypothesis of local isotropy based on the definitions of local homogeneity and isotropy followed by his famous exposition of the turbulence cascade process in a 3/4 page footnote. This hypothesis[2] is precisely about restoring in the statistical sense all the symmetries – except for the scaling one – "in turbulent flow with sufficiently large Reynolds number in sufficiently small regions G of the four-dimensional space (x_1, x_2, x_3, t) not lying close to the boundaries of the flow or other singularities of it". As for the scaling symmetry, Kolmogorov was already aware of the self-similarity before the completion of the first K41 paper,[3] but "did not know how to determine the exponent m" (A. M. Yaglom, *Ann. Rev. Fluid Mech.*, vol. 24, 1994, p. 8). It is Kolmogorov's second similarity hypothesis, which enabled him to find the famous exponent. This shows that Kolmogorov used the term hypotheses of similarity, i.e. scale invariance, intentionally, and avoided use of the term 'universality'. In view of this both the use of the term 'universality assumptions' and the introduction of the term 'universally' in their recast version in the book[4] seem to be unjustified both historically and in essence. The author uses the English translation of the Kolmogorov papers, which in my opinion is a poor translation.[5] This has led to misinterpretations in the book. For example, on p. 77 in footnote 4 Frisch writes: "Just before writing the equivalent of (6.5)[6] Kolmogorov (1941 *c*) uses the following words: *i.e. we may thus assume that*... This can be misread as an additional hypothesis, when actually it is just cautious language meant to bring out that his derivation is not fully rigorous". In the original Russian version it is "t.ye. schitat" meaning 'i.e. to put', which follows directly after the explanation that the viscous term can be neglected for scales large enough in comparison with the Kolmogorov scale, thereby leading ("t.ye. schitat") to the four-fifth law. Of course, this had nothing to do with the rigour of Kolmogorov's derivation.

Chapter 7 is devoted to the phenomenology of turbulence in the sense of Kolmogorov 1941 with some examples of what can be derived by phenomenology. The last section is called 'Beyond phenomenology: finite-time blow-up of ideal flow'. It contains remarks on the controversial issue of formation (or not) of singularities of solutions to the Euler equations in finite time, possibly related to what the author calls depletion of nonlinearity. These subjects are discussed again in the last chapter (9). Intermittency is addressed in chapter 8. It begins with illustrations, definitions and discussion of some experimental material on intermittency which is summarized in the author's usual cautious manner: "It is plausible but not certain that there are intermittency corrections to the K41 theory of the inertial range". This is followed by an exact result for the character of the dependence on order of the even-order scaling exponents: concave and non-decreasing. After this introductory material has been given, various intermittency models based on velocity are presented, ending with the multifractal model and the introduction of the multifractal hypothesis[7] replacing H2.

It is noteworthy that in the probabilistic reformulation (multi) fractality does not require finite-time singularities. The main feature of the multifractal hypothesis is that here the author is back with the universality, postulating two universal exponents and a universal function. Unfortunately, in contrast with K41, the multifractal model (like many other intermittency models) is an arbitrary construction in the sense that it lacks dynamical motivation[8] in general and with respect to the postulated multifractal universality in particular. In the rest of chapter 8 attention is focused on some consequences of multifractal universality based on velocity, multifractal models based on dissipation, bridging between these two, shell models and historical remarks. The last section treats trends in intermittency research and has the character of a review. It is concerned with the role of vortex filaments,[9] with the question whether it is possible to reconcile the multifractal picture (scaling) of turbulence with the geometric picture of vortex filaments, and with PDFs of velocity increments. This concludes the textbook part of the book.

The final chapter (9) is called 'Further reading: a guided tour' and "contains supplementary material beyond the scope of the previous chapters". In fact, chapter 9 has the character of a critical review article on some recent research in the field, and as such is very useful. Nevertheless in spite of feeling guided, the less experienced and especially the beginner will find himself lost in the sea of references, which is only a part of the ocean of publications on turbulence. It seems inappropriate to review a review paper. Therefore only a short description of chapter 9 will be given in view of the number of cross-references between this chapter and chapters 1–8.

After a brief description of some books and review papers on turbulence a review of the following topics is presented: mathematical aspects of fully developed turbulence; dynamical systems, fractals and turbulence; closure, functional and diagrammatic methods; eddy viscosity, multiscale methods an renormalization; and two-dimensional turbulence. The review covers a vast amount of material with a number of useful critical, constructive and historical remarks and suggestions, though few of the themes are marginally relevant to fully developed turbulence. Given the massive nature of the literature on the subject, it is not reasonable to expect an exhaustive review even of the topics selected by the author as those which matter. As in many previous reviews of turbulence research, the characteristic feature of chapter 9 is that it is to a large extent about methods and their failures rather than about results,[10] showing that turbulence remains among the fields with over-production of publications without any real breakthrough in understanding.

The author concludes the chapter (and the book) with the statement[11] that "state-of-the-art experiments and computations are certainly a prerequisite for progress in turbulence", and he is certainly right that "it is a long way from measuring and seeing everything to understanding". The latter, however, requires far more than scaling and phenomenology, since turbulence phenomena are infinitely richer than their manifestation in scaling and related matters. In my view the book puts an excessive emphasis on the latter and reflects the disproportionally large current research effort to study various turbulence exponents without questioning their very existence. In mentioning the difficulties of computations at large Reynolds numbers the author writes: "computers have a long way to go before we can achieve inertial ranges of several decades, as needed for accurate measurements of exponents, while experiments can do this at relatively low cost". The reader would benefit a lot from an explanation of why and to what extent accurate measurement of exponents is important for the understanding of turbulence.[12] Surprisingly the discussion of what matters, but is beyond phenomenology, includes neither vortex stretching/enstrophy generation nor (more generally) the build up of odd moments, with the exception of the four-fifth law and a brief mention of skewness.[13]

Regarding the intended readership of the book, it is said in the Preface that "Primarily, it is intended for those interested in learning about the basics of turbulence or wanting to take a fresh look at the subject". I doubt that those interested in learning about the basics of turbulence can do this solely from a book divorced from the great bulk of experimental information (laboratory, field, numerical). Engineers and experimentalists, looking not only for confirmation of other people's theories, will not find much for them in the book. It is certainly an excellent book for those wanting to take a fresh look at some aspects of the subject. Teachers of turbulence will find it useful, for at least part of the material of their courses. The book is undoubtedly worth reading by everybody concerned with some aspect of turbulence research.

[1] At the end of chapter 2 the reader is promised that the notions of statistical homogeneity and of isotropy will be explained in chapter 3. I did not find them in chapter 3 or elsewhere.

[2] The author omits the presentation of this hypothesis in his book.

[3] Later, in chapter 8 on intermittency, p. 121, the author writes: "A central assumption of the K41 theory in the self-similarity of the random velocity field at inertial-range scale".

[4] In the original version Kolmogorov used the term 'uniquely' only.

[5] I was brought up with the Russian language and have been able to read the original publications.

[6] i.e. four-fifth law.

[7] Possibly motivated, at least in part, by the research in spin glasses by G. Parisi and the author.

[8] Recall the question 'Fractals: Where is the physics?' by L. P. Kadanoff, *Physics Today*, vol. 39, 1986, p. 6.

[9] In proposing for this purpose to use 'faked' flow fields from numerical simulations the author does not mention that such an attempt was made by J. Jimenez, A. A. Wray, P. G. Saffman and R. S. Rogallo 'The structure of intense vorticity in homogeneous isotropic turbulence', *J. Fluid Mech.*, vol. 255, 1993, pp. 65–91.

[10] Recall the paper by H. W. Liepmann, 'The rise and fall of ideas in turbulence', *Am. Scientist*, vol. 67, 1979, pp. 221–228

[11] Note a similar statement by Kolmogorov: "I soon understood that there was little hope of developing a pure, closed theory, and because of absence of such a theory the investigation must be based on hypotheses obtained on processing experimental data" (*Selected Works of A. N. Kolmogorov, I.*, ed. V. M. Tikhomirov, p. 487, Kluwer, 1991).

[12] Without much exaggeration: "The wonderful thing about scaling is that you can get everything right without understanding anything" (R. H. Kraichnan as cited by L. P. Kadanoff 1990, 'Scaling and structures in the hard turbulence region of Rayleigh–Bénard convection', in *New Perspectives in Turbulence*, ed. L. Sirovich, p. 265, Springer).

[13] Hence no reference to the paper by G. I. Taylor, 'Production and dissipation of vorticity in a turbulent fluid,' *Proc. R. Soc. Lond.* A, vol. 164, 1938, pp. 15–23.

A. TSINOBER

Instrumentation for Flows with Combustion. Edited by A. M. K. P. TAYLOR. Academic Press, 1993. 521 pp. ISBN 0-12-683920-4. £70.

'Laser diagnostics have created a revolution in our experimentation in combustion'. This is a quote from R. W. Bilger's chapter 1 entitled 'Basic considerations', and summarizes well the motivation for this book. It contains one chapter on classical 'Physical probes' (chapter 2 by H. A. Becker) and five chapters on non-introdusive laser techniques for the determination of local velocities, density, temperature, the concentration of stable species and radicals. Even the size and concentration of particles can be measured by scattering techniques (chapter 5 by A. R. Jones).

From this list it may seem that nearly everything of interest can be measured in a flame or a technical combustion device but unfortunately this is not true. Some techniques are incompatible with others and the measurement of one quantity may preclude that of another. Particle seeding, which is necessary for velocity measurements using laser Doppler velocimetry (chapter 3 by M. V. Heitor, S. H. Starner, A. M. K. P. Taylor and J. H. Whitelaw), prevents the measurement of density and stable species by Rayleigh or Raman scattering (chapter 7 by M. B. Long). Temperature measurements in practical combustion systems are possible using coherent anti-Stokes Raman spectroscopy (CARS, chapter 4 by L. P. Goss) even if particles or soot are present, but CARS provides only single-point measurements. On the other hand, laser-induced fluorescence is suitable for radicals such as OH, CH and C_2 and its light intensity is strong enough to provide two-dimensional images (chapter 6 by J. M. Seitzmann and R. K. Hanson).

These seven chapters are the contents of the book. They provide a good account of what can be done and what has been done, but only exceptionally do they pose the question of what it would be desirable to measure. Therefore, it is evident that the merit of the book lies in the value of each of the chapter rather than in a unified view of combustion measurement techniques.

In this sense the book has much to offer. Chapter 1 presents an overview of the background literature on measurement techniques and combustion kinetics. Bilger then enters into the ongoing unresolved problems of turbulent combustion and the clarifications that well defined and more refined measurements could provide. Chapter 2 on 'Physical probes' contains examples of probe construction and a detailed physical analysis of quenching in gas sampling probes. While there are ten pages on the less interesting subject of static pressure probes and Pitot probes, there are only one and a half pages (with many references, however) on what is probably the most widely used traditional probe for combustion measurements: the thermocouple.

Laser-Doppler-velocimetry has received various reviews in the past, but the value of chapter 3 lies in the paragraph on simultaneous measurements of velocities and scalars. Here thermocouple measurements for turbulent combustion are discussed in some detail. Mie-scattering and the problem of seeding in flames is addressed in the context of the conserved scalar approach, which is a fundamental concept in non-premixed turbulent combustion. A very useful paragraph in chapter 3 is that on combustion applications, where LDV measurements in laboratory flames, in a model gas turbine combustor, in a kerosine spray and in internal combustion engines are presented.

In chapter 4 the fundamentals of CARS spectroscopy, which has successfully been applied to diatomic gases such as N_2 is first reviewed. Then the optical arrangement, the data acquisition and analysis and the calibration of the instrument are discussed. The most interesting results presented are combined CARS–LDV measurements. Such data provide velocity–scalar correlations and are extremely useful for the development of turbulent combustion models.

Combustion systems very often contain particles – either as fuel (coal particles and droplets) or as combustion-generated particles such as fly ash or soot. The measurement of particle volume fraction, number density and particle size by scattering techniques is addressed in chapter 5. The basic theory is reviewed and examples for measurements of soot, droplets and fly ash are given. This chapter also provides a table that compares the applicability of the different methods in terms of the size range of the particles. Even a list with addresses of manufacturers and distributors is given.

One of the most impressive measurement techniques that can be applied to visualize combustion processes is planar laser-induced fluorescence (PLIF), which is presented in chapter 6. Again the physical principles are briefly reviewed and strategies to obtain quantitative results are discussed. Since LIF is a resonant process, a specific wavelength must be chosen for the particular molecule that one wants to excite with a planar laser sheet. The visualization of OH is now a classical method to visualize the post-flame region where the OH radical is present. Many of the beautiful colour plates in the book were obtained by PLIF of OH. Species commonly used to locate the flame front are CH and C_2. Acetone or biacetyl are sometimes added to the fuel as tracers to visualize the mixing of the fuel in a combustion system. Modern developments aim at NO and O_2 and by multiphoton excitation, at CO, H_2O and the O and H radicals.

The last chapter in this book summarizes the state of the art of non-intrusive multi-dimensional measurement techniques with a review of elastic and inelastic scattering from particles and molecules. The elastic scattering from particles (Lorenz–Mie scattering) provides the highest signal intensity, but since particles must be added to the flow, seeding may be a problem. The technique is most useful for the visualization of flame contours by using particles that are burnt in the flame front. Elastic scattering from molecules (Raleigh scattering) requires the absence of particles and stray light and has been applied successfully to open laboratory flames. Since all molecules in the mixture contribute to the scattered light, additional information such as coupling relations between the concentrations and the temperature must be used to interpret the signal. More specific information about the concentration of chemical species can be obtained by inelastic (Raman) scattering from molecules. Since the scattering Raman cross-section is typically three orders of magnitude lower than the Rayleigh cross-section, the laser intensity must be high and is usually focused into one point for quantitative measurements. Multi-species measurements along a line or tracing the beam inside a multipass cell for one chemical species is a commonly used technique to obtain more than single-point measurements.

Although it comprises contributions from many authors, this book does not suffer from disparity. It rather gains from the up-front expertise of each of the authors in a rapidly developing experimental environment. It has been carefully edited and produced, and represents a very valuable guide-line about the state of the art in laser-diagnostics in combustion.

N. Peters